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MODELING TRANSIENT/STEADY-STATE CONVECTION FROM ISOFLUX VERTICAL PLATES

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ABSTRACT

A new compact analytical model is developed to predict transient heat convection from isoflux vertical plates into a constant property surrounding region. The proposed model is based on a blending of the two asymptotes corresponding to transient diffusion into a half-space and the steady-state convection heat transfer from an isoflux vertical plate. A compact relation for transient and steady-state average Nusselt numbers is proposed. The proposed model is successfully verified with existing experimental data of Goldstein and Eckert. The maximum difference between the proposed compact analytical model and the experimental data is less than 6%.

1. INTRODUCTION

Different heat transfer textbooks present empirical correlations for natural convection heat transfer involving a great variety of geometries [1-3]. These correlations usually account for steady-state processes and comprise cases of either constant surface temperature or constant heat flux. Transient natural convection heat transfer has been studied by several authors. Siegel [4] analyzed the boundary layer growth when a semi-infinite vertical plate was subjected to step heating and described three different conditions in function of time frames. For short periods, the local heat transfer is produced mainly by conduction heat transfer mode, for longer periods steady-state conditions can be applied and for intermediate periods, the local heat transfer coefficient directly depends on time. Goldstein and Eckert [5] validated these results experimentally. Gebhart [6,7] studied the transient regimen for vertical plates and cylinders paying special attention to the thermal capacity

effect of the wall. Churchill [8] proposed a general correlation for free convection from a vertical plate following a step in heat flux density for all times, $Pr > 0.01$ and all Ra in the laminar boundary layer regime. This last correlation is still proposed in practice to be used in heat exchanger design [9]. In recent years, Harris et al. [10] obtained an analytical solution for the transient free convection from a vertical plate when the plate temperature is suddenly changed. Maranzana et al. [11] studied experimentally the transient free convection heat transfer coefficient on a vertical flat plate with different levels of heating immersed in air. Their results showed that for short periods, the transfer coefficient decreases proportionally to t^{-1} and not to $t^{-1/2}$ as the conduction theory had anticipated. De Lorenzo and Padet [12] investigated how free convection heat transfer can be optimized when a periodical heat flux density is applied to a vertical surface. Saeid [13] studied the effect of the periodic oscillation of the surface temperature on transient free convection from a vertical plate. Shapiro and Federovich [14] analyzed the Prandtl number dependence of unsteady natural convection along a vertical plate in a stable stratified fluid. Lin et al. [15] evaluated the transient natural convection boundary layer flow adjacent to a vertical plate heated with a uniform flux in an initially linearly-stratified ambient fluid with Prandtl numbers less than one by using scaling analysis and direct numerical simulation. Fernández-Seara *et al.* [16] investigated transient natural convection heat transfer from a vertical cylindrical tank experimentally, and correlated experimental local convection heat transfer coefficients.

There is no analytical, full-time, and easily implemented solution available for instantaneous heat flow from an isoflux vertical flat plate. This simple solution combines the

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asymptotes corresponding to the short-time half space solution and the long-time steady-state solution to develop a full-time expression.

2. PROBLEM DEFINITION

A vertical plane plate of thickness e , length H and span length L is initially at temperature T_0 in a very large reservoir filled with a fluid at the same temperature. The fluid physical properties (kinematic viscosity ν and thermal diffusivity α) are assumed to be constant.

The plate is suddenly electrically heated with a condition of uniform volumetric heat source. The plate is assumed to be thin so that its thermal inertia is negligible. As a result, heat transfer between the plate and the surrounding fluid is supposed to be at heat flux q'' both constant with time and uniform along the plate. The starting time of heating is chosen as the time origin. The problem is focused on the convective heat transfer during the very first instants of heating. The axes are Ox along the ascending vertical direction, Oy in the direction normal to the plate, and Oz perpendicular to the paper. The plate temperature is denoted $T_w(t)$ at time t . The fluid velocity component parallel to the plate and the fluid temperature are denoted u and T respectively. The flow is assumed to be independent of z .

It is intended to determine the evolution of the plate temperature and heat transfer coefficient as a function of time and the length of time of the unsteady regime at given position x . The physical properties of water are given in Table 1.

Table 1: Physical properties of water used in the analysis

Symbol	Unit	Value
ρ	$kg.m^{-3}$	1000
Pr	-	7
α	$m^2.s^{-1}$	1.40E-07
C_p	$J.kg^{-1}.K^{-1}$	4.18E+03
k_f	$W.m^{-1}.K^{-1}$	0.6
β	K^{-1}	1.80E-04

3. MODEL DEVELOPMENT

A new compact model is developed using two asymptotes, first short-time and second steady-state, assuming the following items;

Assumptions:

- Constant physical properties
- Laminar flow

The methodology is explained in detail in following sections.

3.1 SHORT-TIME ASYMPTOTE

During this initial stage of transfer, the thermal and dynamic problems are uncoupled, contrary to the general case of natural convection. It is therefore possible to first solve the thermal problem, and, in a second step, the temperature field being calculated, to solve the dynamical problem. It is also worth noting that the heat transfer problem is that of a stepwise variation of heat flux on a semi-infinite solid, whose solution is well known. Eckert and Drake [17] solved this problem and calculated the value of Nusselt number.

$$h(t) = \frac{\sqrt{\pi}}{2} \frac{k}{\sqrt{\alpha t}} = 0.886 \frac{k}{\sqrt{\alpha t}} \quad (1)$$

where α is thermal diffusivity, k is thermal conductivity, t is time, and h is convection heat transfer coefficient, respectively. Therefore for Nusselt number we can write;

$$Nu(t) = \frac{h(t)x}{K} = 0.886 \frac{x}{\sqrt{\alpha t}} \quad (2)$$

Integrating over the length of plate, we can derive the equation for average Nusselt number.

$$\overline{Nu}_H(t) = \frac{1}{H} \int_0^H Nu(x) dx = 0.443 \frac{H}{\sqrt{\alpha t}} \quad (3)$$

3.2. STEADY-STATE ASYMPTOTE

Steady-state convection heat transfer from a vertical plate is investigated analytically in [3], and the result can be shown as;

$$Nu(x) = \frac{q''}{T_0(x) - T_\infty} \frac{x}{k} = h(x) \frac{x}{k} = \frac{2}{360^{\frac{1}{5}}} \left(\frac{Pr}{4} \right)^{\frac{1}{5}} Ra_{*x}^{\frac{1}{5}} \quad (4)$$

Where Ra_{*x} is Rayleigh number and can be defined as;

$$Ra_{*x} = \frac{g\beta x^4 q''}{\alpha \nu k} \quad (5)$$

where g is acceleration due to gravity, β is thermal expansion coefficient, and ν is kinematic viscosity, respectively. In a same way, for average Nusselt number we will have;

$$\overline{Nu}_H = \frac{1}{H} \int_0^H Nu(x) dx = \frac{5}{9} \frac{2}{360^{\frac{1}{5}}} \left(\frac{Pr}{4} \right)^{\frac{1}{5}} \left(\frac{g\beta q''}{\alpha \nu k} \right)^{\frac{1}{5}} H^{\frac{4}{5}} \quad (6)$$

Figures 1 and 2 show these two asymptotes, plotted for two different values of q'' , for both local and average Nusselt numbers respectively.

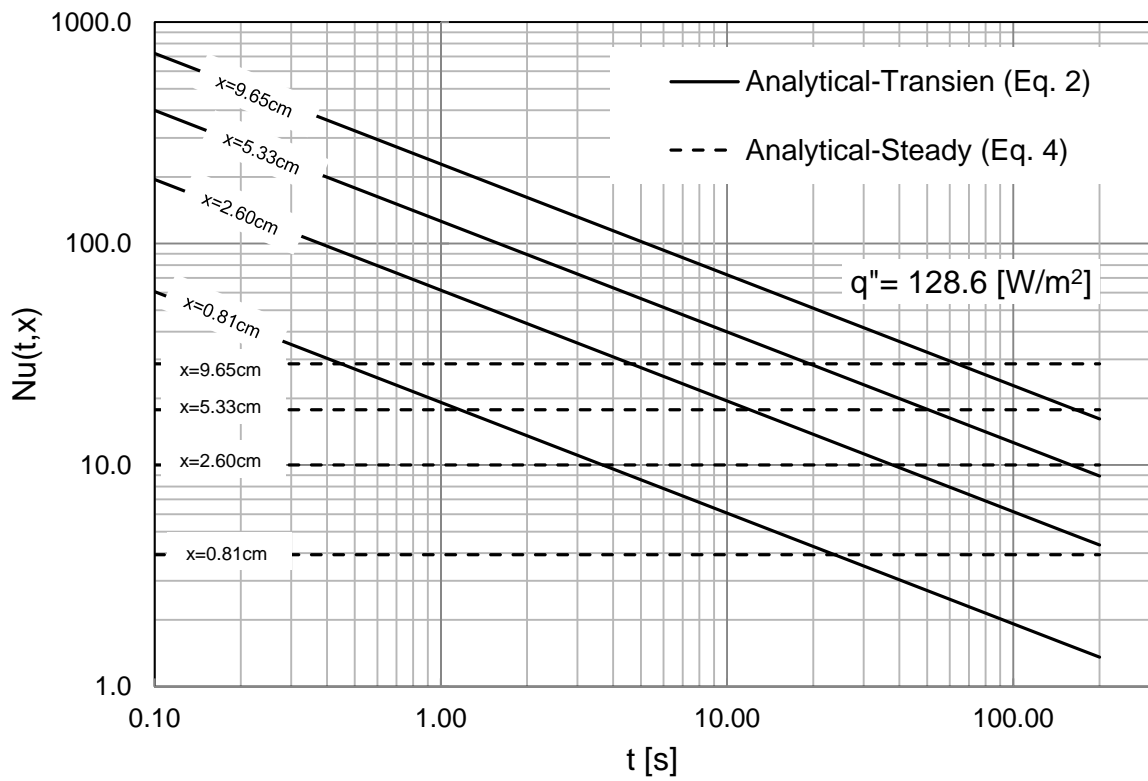
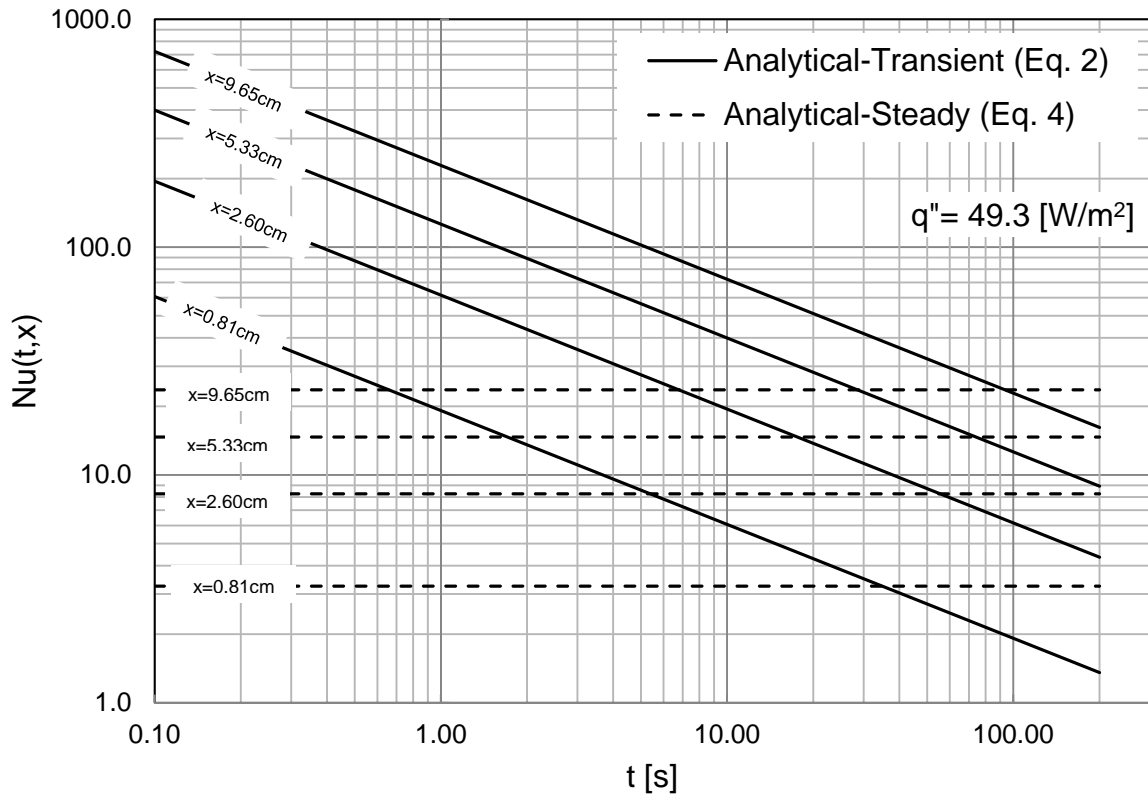


Fig. 1: Analytical solutions for local Nusselt number for two values of q'' .

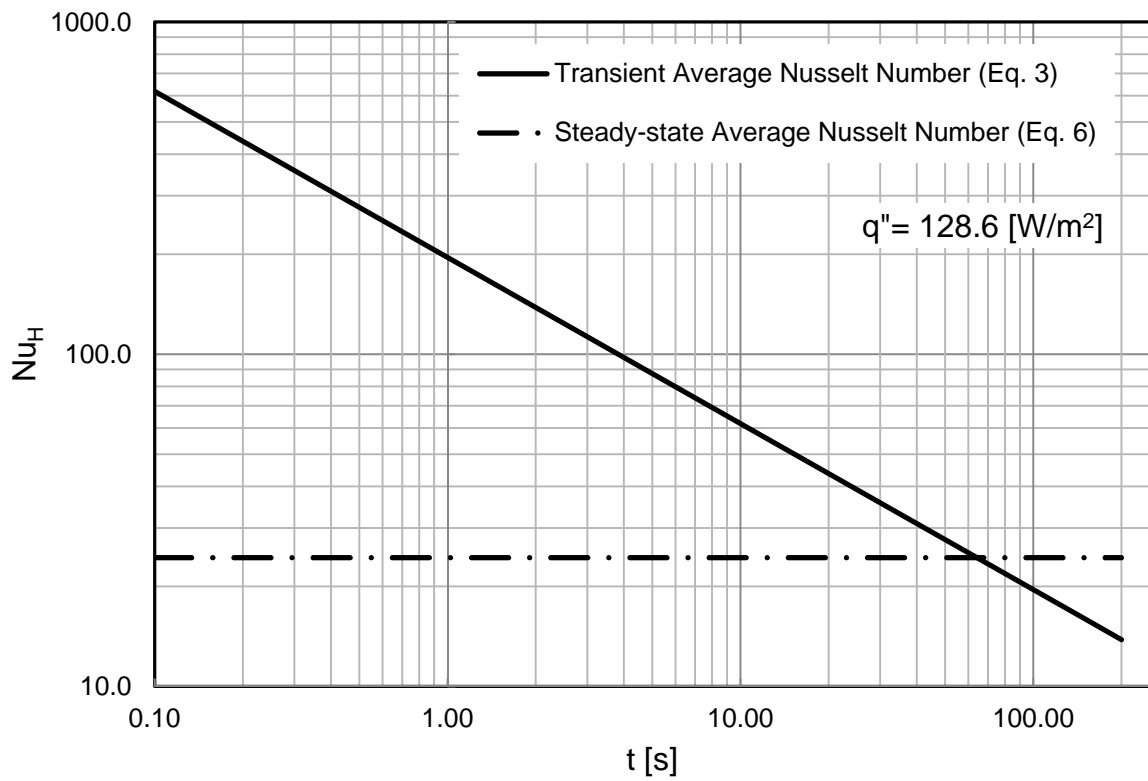
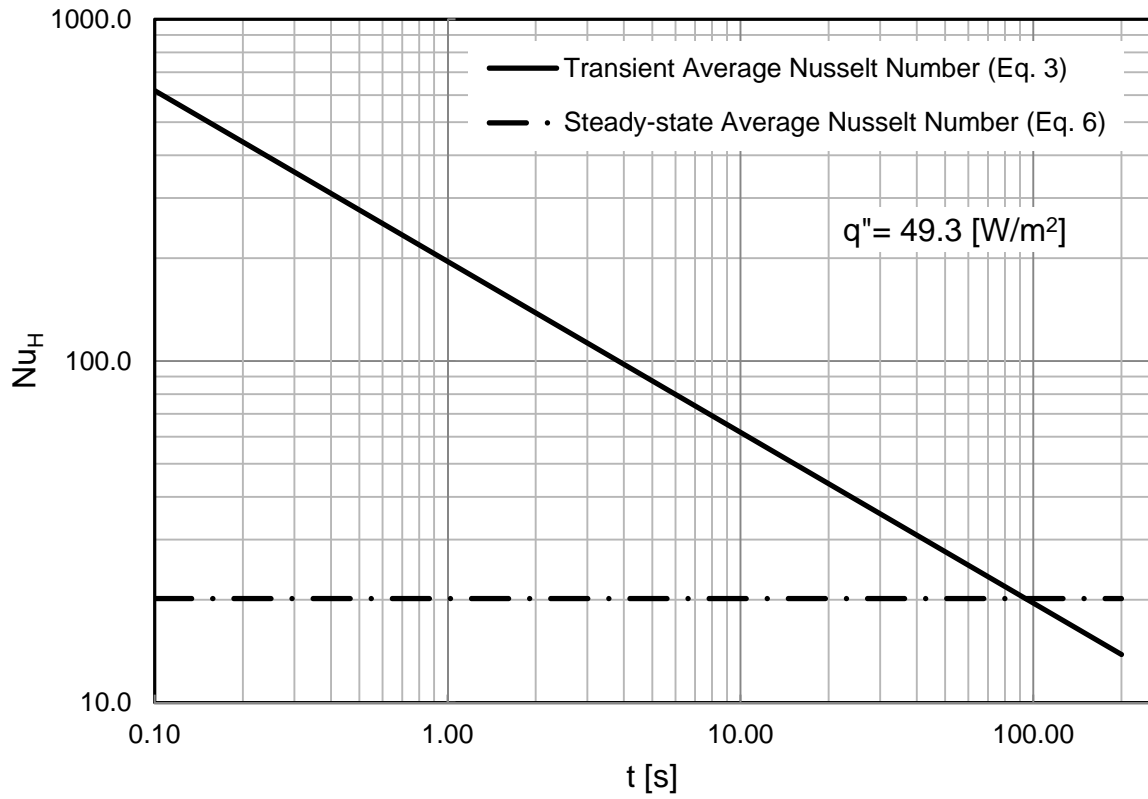


Fig. 2: Analytical solutions for average Nusselt number for two values of q'' .

3.3. MODEL VERIFICATION

Goldstein and Eckert [5] used a Mach-Zehnder interferometer setup and performed a series of experiments on free convection heat transfer from an isoflux vertical flat plate immersed in a constant temperature water reservoir. They

reported local convective heat transfer coefficient in their study. Figure 3 shows the local heat transfer coefficient based on the data they reported, for two values of heat fluxes from the plate.

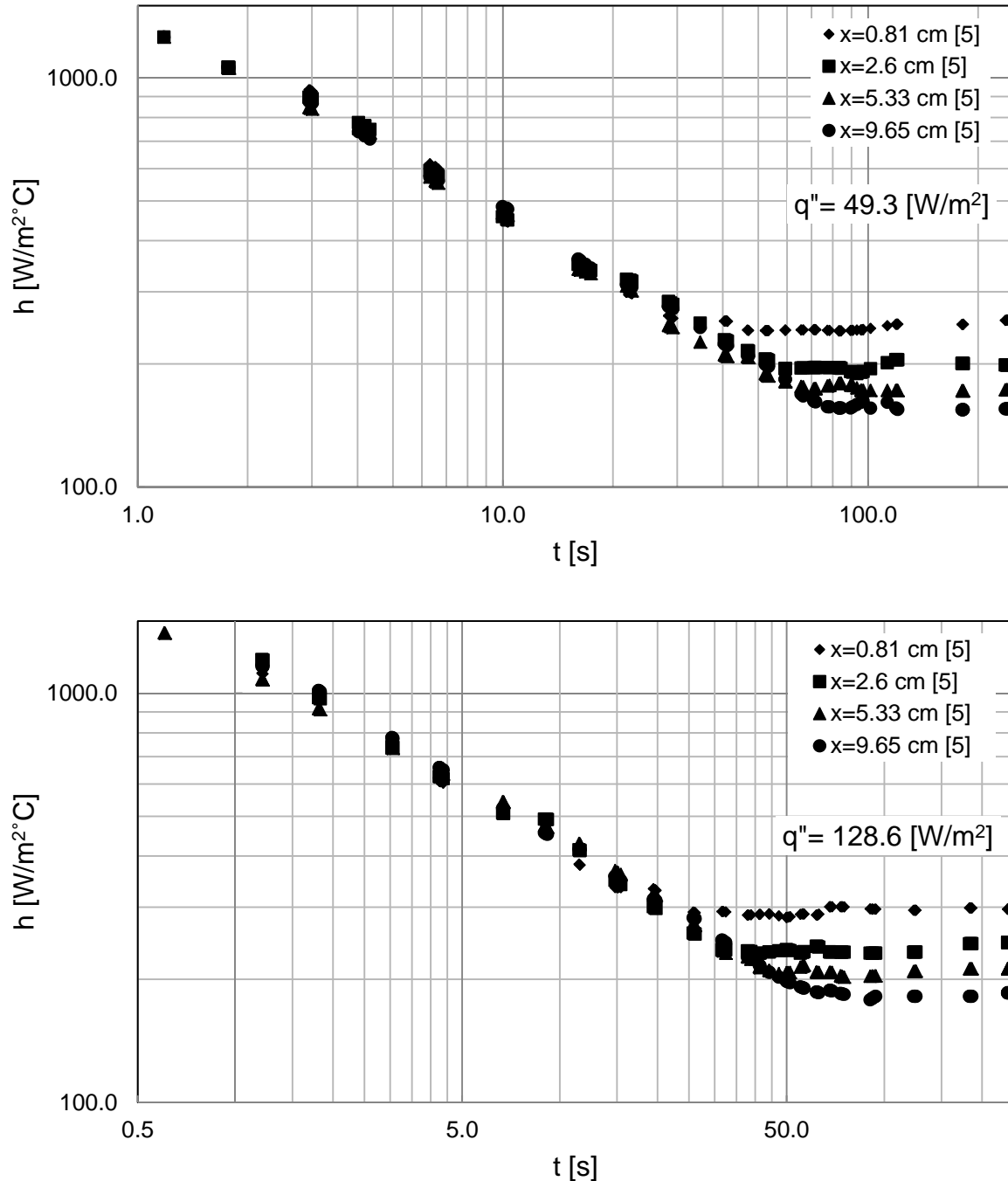


Fig. 3: Experimental results for local Nusselt number for two values of q'' .

Experimental and analytical data are compared in Fig. 4. As it can be seen there is a perfect agreement between the

analytical solution and experimental data, for both transient and steady regimes.

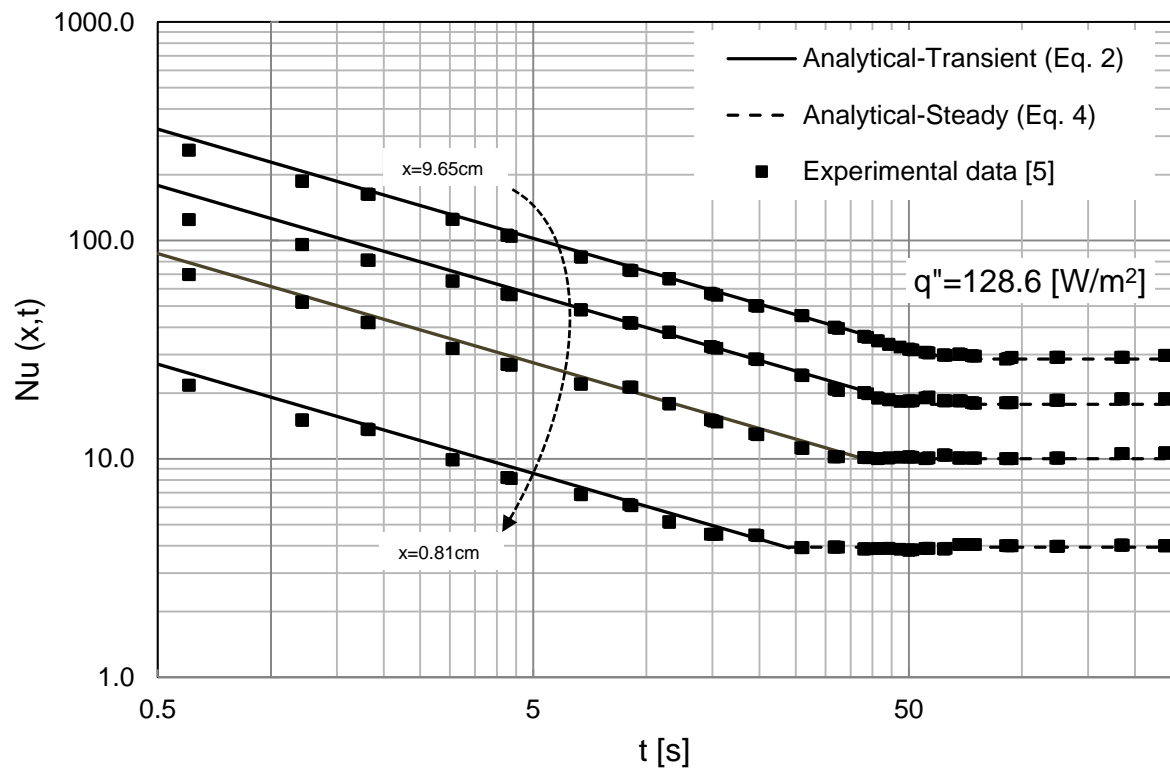
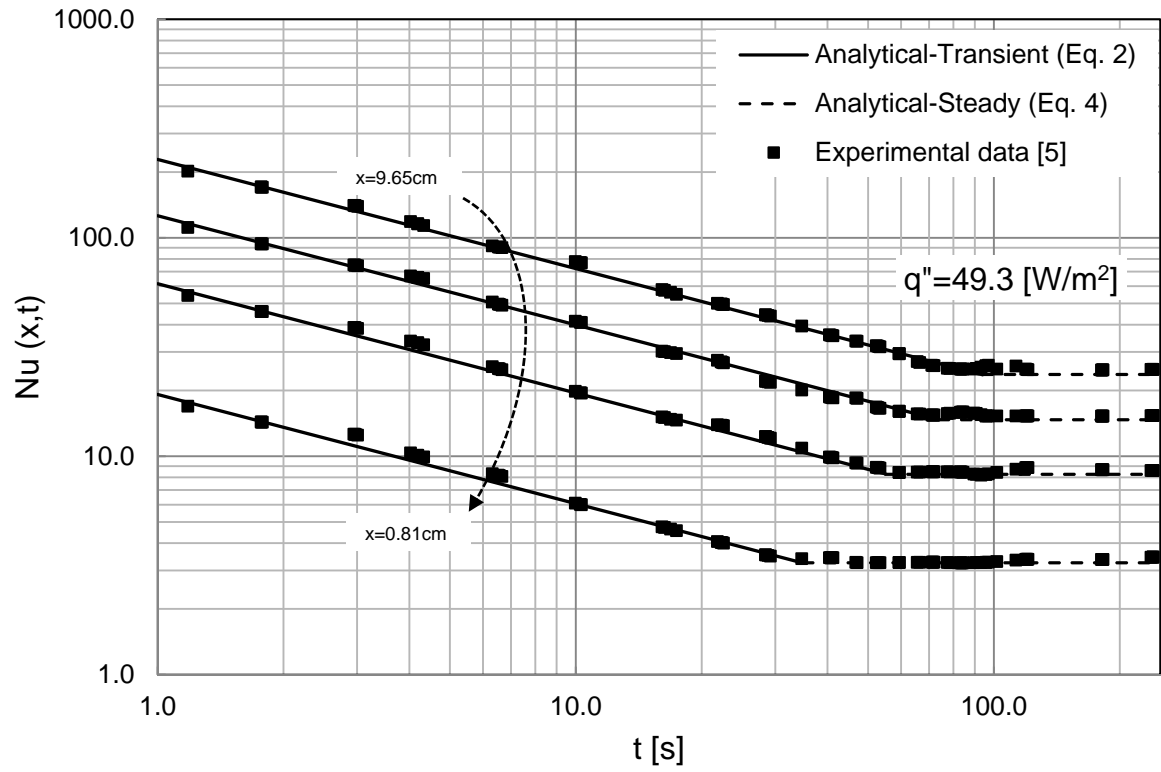


Fig. 4: Comparison of local Nusselt number for analytical result and experimental data for two values of q'' .

3.4. FULL-TIME MODEL

To combine the asymptotic solutions, the short and long time limits, the composite solution technique of Churchill and Usagi [18] is used;

$$Nu^*(t) = [\overline{Nu}_H(t)^n + \overline{Nu}_H^n]^{1/n} \quad (7)$$

A fitting parameter of $n = 10$ was found to minimize the maximum percent difference between the model and the exact solution in the comparison shown in Fig. 5. So the model proposed in this work for dimensionless convective heat flow rate from an isoflux vertical flat plate, for the full range of time can be summarized as follows:

$$\begin{aligned} Nu^*(t) &= [\overline{Nu}_H(t)^{10} + \overline{Nu}_H^{10}]^{0.1} \\ &= [(0.443 \frac{H}{\sqrt{at}})^{10} \\ &+ (\frac{5}{9} \frac{2}{360^{1/5}} \left(\frac{Pr}{4 + Pr} \right)^{1/5} \left(\frac{g\beta q''}{\alpha \nu k} \right)^{1/5} H^{4/5})^{10}]^{0.1} \end{aligned} \quad (8)$$

3.5. Model Validation

The proposed models for dimensionless transient heat flow rate from Eq. (8) is validated using the verified exact solution for isoflux vertical plate immersed in water, and the result are shown in Fig. 5. As it can be seen there is a perfect agreement between results from Eq. 8 and exact solution. The maximum error is less than 6%, and it occurs in the transition region from transient to steady-state.

4. CONCLUSION AND SUMMARY

A new compact analytical model was developed to predict full-range-time heat convection from isoflux vertical plates into a constant property surrounding region, based on a blending of the two asymptotes corresponding to transient diffusion into a half-space and the steady-state convection heat transfer from an isoflux vertical plate. The results can be highlighted as bellow;

- A compact relation for transient and steady-state average Nusselt numbers was proposed.
- The model was verified using analytical solutions from [3] and [17].

The analytical solution was verified before by experimental data from [5].

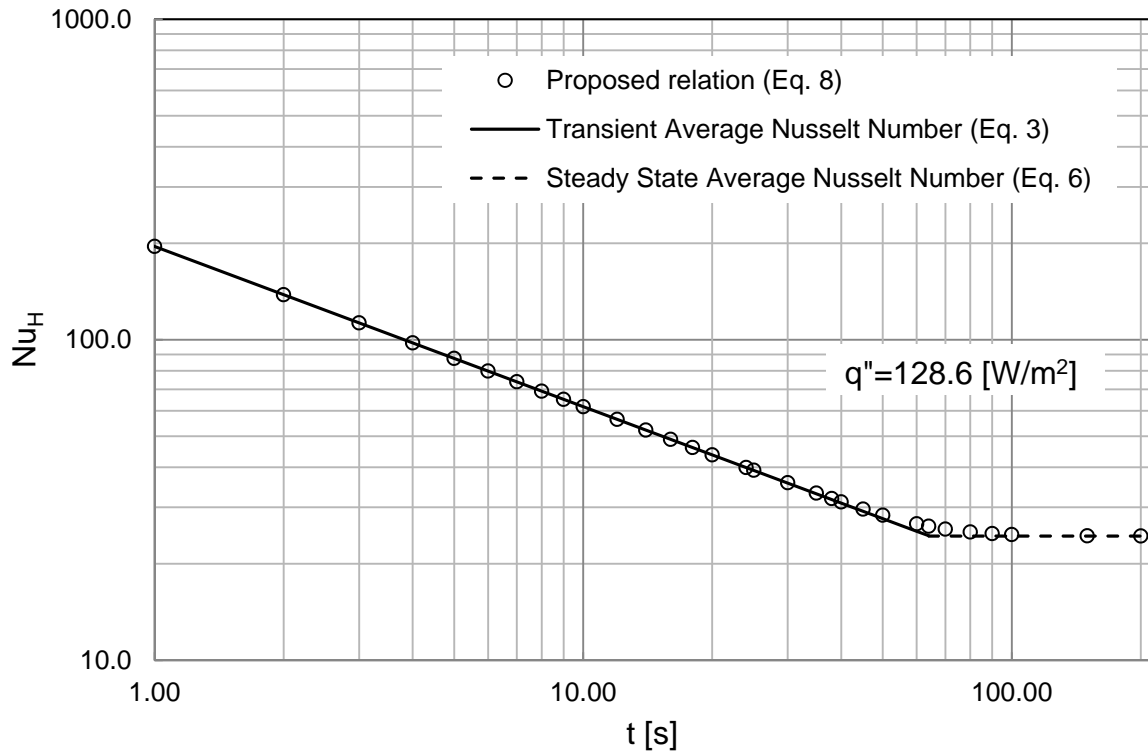
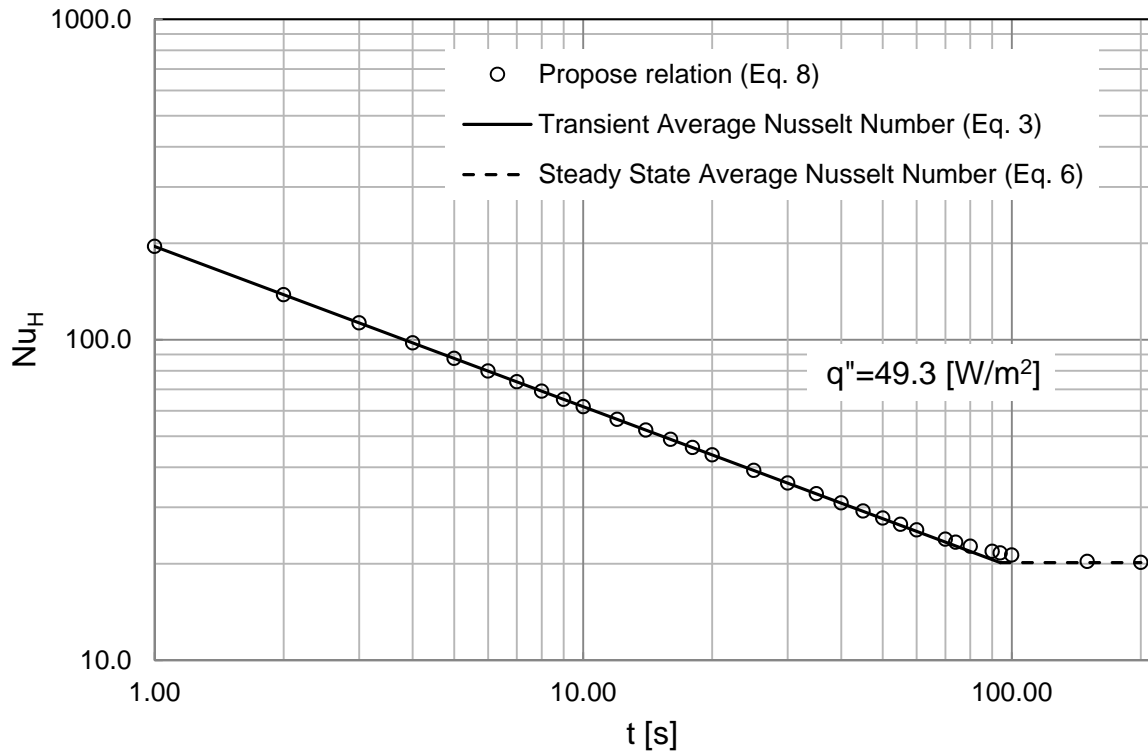


Fig. 5: Validation of the proposed relation with exact solution for two values of q'' .

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